

# Human review packet: GJ18InformativeRatingSystems

Generated from byte-pinned source excerpts, the expanded PaperInterface specifications, and hash-bound raw-source-to-Spec screening records.

**Paper:** GJ18InformativeRatingSystems

**Paper id:** GJ18InformativeRatingSystems

**Source version:** Byte-pinned arXiv 1810.13028 TeX plus the finite-chain corrected model recorded  
2026-07-24

**Source record:** audit/paper\_statement\_map.json

**Rows:** 3

**Generated:** 2026-09-06

**Source URL:** [official source](#)

## How to use this packet

For each source claim, compare the exact source excerpts with the expanded PaperInterface specification. Mark up this PDF or fill the blank reviewer box in the TeX source; return the annotated file for reconciliation into the paper-local human-review log.

## Open the interactive dashboard

The interactive dashboard is an optional alternative to using this PDF; it presents the same review surface rather than an additional review requirement. After forking and cloning (or pulling) AppliedModelingLib, run the following from the repository root. The cache command is needed only the first time in a new clone.

```
cd <your-repository-checkout>
lake exe cache get
python3 scripts/review_dashboard.py --paper GJ18InformativeRatingSystems --serve
```

Open <http://127.0.0.1:8765/> in a browser and keep that terminal running while reviewing. The dashboard records saved annotations in this paper's local reviewer-owned review record.

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# Governing Lean declarations

These exact graph-bound declaration bodies are retained by the displayed specifications or reviewed prerequisites. They are supporting context and do not add source-result rows or semantic judgments.

## AppliedModelingLib.Probability.FiniteRatingLDPModel

**Declaration location:** reusable library

### Lean declaration body

```
field typeLaw:
{θ : Type u_2} →
{Rating : Type u_3} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPModel θ Rating → θ → PMF Rating

field score:
{θ : Type u_2} →
{Rating : Type u_3} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPModel θ Rating → Rating → ℝ
```

## AppliedModelingLib.Probability.finiteIdScoreSum

**Declaration location:** reusable library

### Lean declaration body

```
type:
{α : Type u_1} → {ι : Type u_2} → [Fintype ι] → (α → ℝ) → (ι → α) → ℝ

value:
fun {α : Type u_1} {ι : Type u_2} [Fintype ι] (score : α → ℝ) (sample : ι → α) => ∑ i : ι, score (sample i)
```

## AppliedModelingLib.Probability.finiteMGF

**Declaration location:** reusable library

### Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (score : α → ℝ) (z : ℝ) => ∑ a : α, (μ a).toReal * Real.exp (z * score a)
```

## AppliedModelingLib.Probability.finiteLogMGF

**Declaration location:** reusable library

### Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (score : α → ℝ) (z : ℝ) =>
Real.log (AppliedModelingLib.Probability.finiteMGF μ score z)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.finiteMGF](#)

## AppliedModelingLib.Probability.FiniteRatingLDPModel.logMGF

**Declaration location:** reusable library

## Lean declaration body

```
type:
{θ : Type u_2} →
{Rating : Type u_3} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating → θ → ℝ → ℝ

value:
fun {θ : Type u_2} {Rating : Type u_3} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating) (t : θ) (z : ℝ) =>
AppliedModelingLib.Probability.finiteLogMGF (M.typeLaw t) M.score z
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteLogMGF](#)

## AppliedModelingLib.Probability.finiteLegendreValue

**Declaration location:** reusable library

## Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ → ℝ → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (score : α → ℝ) (a z : ℝ) =>
z * a - AppliedModelingLib.Probability.finiteLogMGF μ score z
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.finiteLogMGF](#)

## AppliedModelingLib.Probability.finiteRateFunction

**Declaration location:** reusable library

## Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (score : α → ℝ) (a : ℝ) =>
sSup (Set.range fun (z : ℝ) => AppliedModelingLib.Probability.finiteLegendreValue μ score a z)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.finiteLegendreValue](#)

## AppliedModelingLib.Probability.FiniteRatingLDPMModel.rateFunction

**Declaration location:** reusable library

## Lean declaration body

```
type:
{θ : Type u_2} →
{Rating : Type u_3} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating → θ → ℝ → ℝ

value:
fun {θ : Type u_2} {Rating : Type u_3} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating) (t : θ) (a : ℝ) =>
AppliedModelingLib.Probability.finiteRateFunction (M.typeLaw t) M.score a
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteRateFunction](#)

## AppliedModelingLib.Probability.finiteRateFunctionTop

**Declaration location:** reusable library

### Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ → WithTop ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (score : α → ℝ) (a : ℝ) =>
  sSup (Set.range fun (z : ℝ) => ↑(AppliedModelingLib.Probability.finiteLegendreValue μ score a z))
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.finiteLegendreValue](#)

## AppliedModelingLib.Probability.FiniteRatingLDPMModel.rateFunctionTop

**Declaration location:** reusable library

### Lean declaration body

```
type:
{θ : Type u_2} →
  {Rating : Type u_3} →
  [inst : Fintype Rating] →
  [inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating → θ → ℝ → WithTop ℝ

value:
fun {θ : Type u_2} {Rating : Type u_3} [Fintype Rating] [DecidableEq Rating]
  (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating) (t : θ) (a : ℝ) =>
  AppliedModelingLib.Probability.finiteRateFunctionTop (M.typeLaw t) M.score a
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteRateFunctionTop](#)

## AppliedModelingLib.Probability.finiteChainJointFloorRatingSample

**Declaration location:** reusable library

### Lean declaration body

```
type:
{n : ℕ} → Type u_1 → (Fin n → ℝ) → ℕ → Type u_1

value:
fun {n : ℕ} (Rating : Type u_1) (sampleRate : Fin n → ℝ) (k : ℕ) =>
  (θ : Fin n) → Fin (AppliedModelingLib.Probability.floorSampleCount sampleRate θ k) → Rating
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.floorSampleCount](#)

## AppliedModelingLib.Probability.logDecay

**Declaration location:** reusable library

### Lean declaration body

```
type:
(ℕ → ℝ) → ℕ → ℝ

value:
fun (p : ℕ → ℝ) (n : ℕ) => -(↑n)-1 * Real.log (p n)
```

## AppliedModelingLib.Probability.HasExponentialRate

**Declaration location:** reusable library

## Lean declaration body

```
type:
( $\mathbb{N} \rightarrow \mathbb{R}$ )  $\rightarrow \mathbb{R} \rightarrow \text{Prop}$ 

value:
fun (p :  $\mathbb{N} \rightarrow \mathbb{R}$ ) (rate :  $\mathbb{R}$ ) => Filter.Tendsto (AppliedModelingLib.Probability.logDecay p) Filter.atTop (nhds rate)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.logDecay](#)

## AppliedModelingLib.Probability.ExponentialRateCertificate

**Declaration location:** reusable library

## Lean declaration body

```
field eventually_pos:
 $\forall \{p : \mathbb{N} \rightarrow \mathbb{R}\} \{rate : \mathbb{R}\},$ 
  AppliedModelingLib.Probability.ExponentialRateCertificate p rate  $\rightarrow \forall (n : \mathbb{N})$  in Filter.atTop,  $0 < p\ n$ 

field has_rate:
 $\forall \{p : \mathbb{N} \rightarrow \mathbb{R}\} \{rate : \mathbb{R}\},$ 
  AppliedModelingLib.Probability.ExponentialRateCertificate p rate  $\rightarrow$ 
    AppliedModelingLib.Probability.HasExponentialRate p rate
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.HasExponentialRate](#)

## AppliedModelingLib.Probability.withTopRealScale

**Declaration location:** reusable library

## Lean declaration body

```
type:
 $\mathbb{R} \rightarrow \text{WithTop } \mathbb{R} \rightarrow \text{WithTop } \mathbb{R}$ 

value:
fun (c :  $\mathbb{R}$ ) (x : WithTop  $\mathbb{R}$ ) => WithTop.map (fun (r :  $\mathbb{R}$ ) => c * r) x
```

## AppliedModelingLib.Probability.FiniteRatingLDPModel.pairwiseRateObjectiveTop

**Declaration location:** reusable library

## Lean declaration body

```
type:
{ $\theta$  : Type u_2}  $\rightarrow$ 
  {Rating : Type u_3}  $\rightarrow$ 
    [inst : Fintype Rating]  $\rightarrow$ 
      [inst_1 : DecidableEq Rating]  $\rightarrow$ 
        AppliedModelingLib.Probability.FiniteRatingLDPModel  $\theta$  Rating  $\rightarrow (\theta \rightarrow \mathbb{R}) \rightarrow \theta \rightarrow \theta \rightarrow \mathbb{R} \rightarrow \text{WithTop } \mathbb{R}$ 

value:
fun { $\theta$  : Type u_2} {Rating : Type u_3} [Fintype Rating] [DecidableEq Rating]
  (M : AppliedModelingLib.Probability.FiniteRatingLDPModel  $\theta$  Rating) (sampleRate :  $\theta \rightarrow \mathbb{R}$ ) (hi lo :  $\theta$ ) (a :  $\mathbb{R}$ ) =>
  AppliedModelingLib.Probability.withTopRealScale (sampleRate hi) (M.rateFunctionTop hi a) +
  AppliedModelingLib.Probability.withTopRealScale (sampleRate lo) (M.rateFunctionTop lo a)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPModel.rateFunctionTop](#)
- [AppliedModelingLib.Probability.withTopRealScale](#)

## AppliedModelingLib.Probability.FiniteRatingLDPModel.pairwiseThresholdRateTop

**Declaration location:** reusable library

## Lean declaration body

```
type:
{θ : Type u_2} →
{Rating : Type u_3} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating → (θ → ℝ) → θ → θ → WithTop ℝ

value:
fun {θ : Type u_2} {Rating : Type u_3} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel θ Rating) (sampleRate : θ → ℝ) (hi lo : θ) =>
  snf (Set.range fun (a : ℝ) => M.pairwiseRateObjectiveTop sampleRate hi lo a)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel.pairwiseRateObjectiveTop](#)

## AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop

**Declaration location:** reusable library

## Lean declaration body

```
type:
{Seller : Type u_1} →
{Rating : Type u_2} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating → (Seller → ℝ) → Seller → Seller → WithTop ℝ

value:
fun {Seller : Type u_1} {Rating : Type u_2} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating) (sampleRate : Seller → ℝ)
(hi lo : Seller) =>
  M.pairwiseThresholdRateTop sampleRate hi lo
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel.pairwiseThresholdRateTop](#)

## AppliedModelingLib.pmfExp

**Declaration location:** reusable library

## Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (α → ℝ) → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (f : α → ℝ) => ∑ a : α, (μ a).toReal * f a
```

## AppliedModelingLib.pmfPi

**Declaration location:** reusable library

## Lean declaration body

```
type:
{ι : Type u_1} →
[Fintype ι] →
[DecidableEq ι] → {α : ι → Type u_2} → [(i : ι) → Fintype (α i)] → ((i : ι) → PMF (α i)) → PMF ((i : ι) → α i)

value:
fun {ι : Type u_1} [Fintype ι] [DecidableEq ι] {α : ι → Type u_2} [(i : ι) → Fintype (α i)] (μ : (i : ι) → PMF (α i)) =>
  PMF.ofFintype (fun (f : (i : ι) → α i) => ∏ i : ι, (μ i) (f i)) (AppliedModelingLib.pmfPi._proof_1 μ)
```

## AppliedModelingLib.pmfProb

**Declaration location:** reusable library

## Lean declaration body

```
type:
{α : Type u_1} → [Fintype α] → PMF α → (p : α → Prop) → [DecidablePred p] → ℝ

value:
fun {α : Type u_1} [Fintype α] (μ : PMF α) (p : α → Prop) [DecidablePred p] =>
  AppliedModelingLib.pmfExp μ fun (a : α) => if p a then 1 else 0
```

## Direct retained dependencies

- [AppliedModelingLib.pmfExp](#)

## AppliedModelingLib.pmfProduct

**Declaration location:** reusable library

## Lean declaration body

```
type:
(ι : Type u_1) → (α : Type u_2) → [Fintype ι] → [DecidableEq ι] → [Fintype α] → PMF α → PMF (ι → α)

value:
fun (ι : Type u_1) (α : Type u_2) [Fintype ι] [DecidableEq ι] [Fintype α] (μ : PMF α) =>
  PMF.ofFintype (fun (f : ι → α) => [! i : ι, μ (f i)] (AppliedModelingLib.pmfProduct._proof_1 ι α μ))
```

## AppliedModelingLib.Probability.finiteChainJointFloorRatingLaw

**Declaration location:** reusable library

## Lean declaration body

```
type:
{n : ℕ} →
  {Rating : Type u_1} →
  [inst : Fintype Rating] →
  [inst_1 : DecidableEq Rating] →
  AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
  (sampleRate : Fin n → ℝ) →
  (k : ℕ) → PMF (AppliedModelingLib.Probability.finiteChainJointFloorRatingSample Rating sampleRate k)

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
  (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ) (k : ℕ) =>
  AppliedModelingLib.pmfPi fun (θ : Fin n) =>
    AppliedModelingLib.pmfProduct (Fin (AppliedModelingLib.Probability.floorSampleCount sampleRate θ k)) Rating
    (M.typeLaw θ)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteChainJointFloorRatingSample](#)
- [AppliedModelingLib.Probability.floorSampleCount](#)
- [AppliedModelingLib.pmfPi](#)
- [AppliedModelingLib.pmfProduct](#)

## AppliedModelingLib.uniformPMF

**Declaration location:** reusable library

## Lean declaration body

```
type:
(α : Type u_1) → [Fintype α] → [Nonempty α] → PMF α

value:
fun (α : Type u_1) [Fintype α] [Nonempty α] =>
  PMF.ofFintype (fun (x : α) => (↑ (Fintype.card α))-1) (AppliedModelingLib.uniformPMF._proof_1 α)
```

## GJ18InformativeRatingSystems.clarifiedMatchRateBounds

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → (Fin n → ℝ) → Prop

value:
fun {n : ℕ} (sampleRate : Fin n → ℝ) => ∀ (theta : Fin n), 0 < sampleRate theta ∧ sampleRate theta ≤ 1
```

## GJ18InformativeRatingSystems.clarifiedNontrivialRatingScale

**Declaration location:** paper-local

## Lean declaration body

```
type:
ℕ → Prop

value:
fun (m : ℕ) => 1 ≤ m
```

## GJ18InformativeRatingSystems.clarifiedUniformTypePrior

**Declaration location:** paper-local

## Lean declaration body

```
type:
(n : ℕ) → 0 < n → PMF (Fin n)

value:
fun (n : ℕ) (hn : 0 < n) => AppliedModelingLib.uniformPMF (Fin n)
```

## Direct retained dependencies

- [AppliedModelingLib.uniformPMF](#)

## GJ18InformativeRatingSystems.finiteChainAdjacentIndex

**Declaration location:** paper-local

## Lean declaration body

```
type:
ℕ → Type

value:
fun (n : ℕ) => { i : Fin n // ↑i + 1 < n }
```

## GJ18InformativeRatingSystems.finiteChainOrderedPair

**Declaration location:** paper-local

## Lean declaration body

```
type:
ℕ → Type

value:
fun (n : ℕ) => { p : Fin n × Fin n // ↑p.2 < ↑p.1 }
```

## GJ18InformativeRatingSystems.finiteChainAdjacentPair

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
  GJ18InformativeRatingSystems.finiteChainAdjacentIndex n → GJ18InformativeRatingSystems.finiteChainOrderedPair n

value:
fun {n : ℕ} (i : GJ18InformativeRatingSystems.finiteChainAdjacentIndex n) =>
  (((↑↑i + 1, GJ18InformativeRatingSystems.finiteChainAdjacentPair._proof_1 i), ↑i),
   GJ18InformativeRatingSystems.finiteChainAdjacentPair._proof_2 i)
```

## Direct retained dependencies

- [GJ18InformativeRatingSystems.finiteChainAdjacentIndex](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)

## GJ18InformativeRatingSystems.finiteChainOrderedPairHi

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → GJ18InformativeRatingSystems.finiteChainOrderedPair n → Fin n

value:
fun {n : ℕ} (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) => (↑p).1
```

## Direct retained dependencies

- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)

## GJ18InformativeRatingSystems.finiteChainOrderedPairLo

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → GJ18InformativeRatingSystems.finiteChainOrderedPair n → Fin n

value:
fun {n : ℕ} (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) => (↑p).2
```

## Direct retained dependencies

- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)

## GJ18InformativeRatingSystems.finiteChainOrderedPairThresholdRateTop

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
  {Rating : Type u_1} →
  [inst : Fintype Rating] →
  [inst_1 : DecidableEq Rating] →
  AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
  (Fin n → ℝ) → GJ18InformativeRatingSystems.finiteChainOrderedPair n → WithTop ℝ

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
  (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ)
  (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) =>
  AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop M sampleRate
  (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p) (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)

## GJ18InformativeRatingSystems.finiteChainAdjacentThresholdRateTop

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
{Rating : Type u_1} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
(Fin n → ℝ) → GJ18InformativeRatingSystems.finiteChainAdjacentIndex n → WithTop ℝ

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ)
(i : GJ18InformativeRatingSystems.finiteChainAdjacentIndex n) =>
GJ18InformativeRatingSystems.finiteChainOrderedPairThresholdRateTop M sampleRate
(GJ18InformativeRatingSystems.finiteChainAdjacentPair i)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.finiteChainAdjacentIndex](#)
- [GJ18InformativeRatingSystems.finiteChainAdjacentPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairThresholdRateTop](#)

## GJ18InformativeRatingSystems.iidFloorSourceStateCarrier

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → Type u_1 → (Fin n → ℝ) → ℕ → Type u_1

value:
fun {n : ℕ} (Rating : Type u_1) (sampleRate : Fin n → ℝ) (k : ℕ) =>
AppliedModelingLib.Probability.finiteChainJointFloorRatingSample Rating sampleRate k
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.finiteChainJointFloorRatingSample](#)

## GJ18InformativeRatingSystems.iidFloorSourceStateLaw

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
{Rating : Type u_1} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
(sampleRate : Fin n → ℝ) →
(k : ℕ) → PMF (GJ18InformativeRatingSystems.iidFloorSourceStateCarrier Rating sampleRate k)

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ) (k : ℕ) =>
AppliedModelingLib.Probability.finiteChainJointFloorRatingLaw M sampleRate k
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteChainJointFloorRatingLaw](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)

## GJ18InformativeRatingSystems.iidFloorSourceStateScore

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
{Rating : Type u_1} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
(sampleRate : Fin n → ℝ) →
(k : ℕ) → GJ18InformativeRatingSystems.iidFloorSourceStateCarrier Rating sampleRate k → Fin n → ℝ

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ) (k : ℕ)
(a : GJ18InformativeRatingSystems.iidFloorSourceStateCarrier Rating sampleRate k) (a_1 : Fin n) =>
AppliedModelingLib.Probability.finiteChainJointFloorAverageScore M sampleRate k a a_1
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteChainJointFloorAverageScore](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)

## GJ18InformativeRatingSystems.minFiniteChainAdjacentThresholdRateTop

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
{Rating : Type u_1} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
[Nonempty (GJ18InformativeRatingSystems.finiteChainAdjacentIndex n)] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating → (Fin n → ℝ) → WithTop ℝ

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
[Nonempty (GJ18InformativeRatingSystems.finiteChainAdjacentIndex n)]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ) =>
Finset.univ.inf' GJ18InformativeRatingSystems.minFiniteChainAdjacentThresholdRate._proof_1
(GJ18InformativeRatingSystems.finiteChainAdjacentThresholdRateTop M sampleRate)
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.finiteChainAdjacentIndex](#)
- [GJ18InformativeRatingSystems.finiteChainAdjacentThresholdRateTop](#)

## GJ18InformativeRatingSystems.sourceOrdinalUpperTail

**Declaration location:** paper-local

## Lean declaration body

```
type:
{Seller : Type u_1} →
{m : ℕ} → AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller (Fin (m + 1)) → Seller → Fin (m + 1) → ℝ

value:
fun {Seller : Type u_1} {m : ℕ} (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller (Fin (m + 1)))
(θ : Seller) (t : Fin (m + 1)) =>
AppliedModelingLib.pmfProb (M.typeLaw θ) fun (y : Fin (m + 1)) => t ≤ y
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.pmfProb](#)

## GJ18InformativeRatingSystems.sourceStateScoreGapLeftTailProb

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} →
{Omega : Type u_1} →
[Fintype Omega] →
[DecidableEq Omega] → PMF Omega → (Omega → Fin n → ℝ) → GJ18InformativeRatingSystems.finiteChainOrderedPair n → ℝ

value:
fun {n : ℕ} {Omega : Type u_1} [Fintype Omega] [DecidableEq Omega] (mu : PMF Omega) (score : Omega → Fin n → ℝ)
(p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) =>
AppliedModelingLib.pmfProb mu fun (omega : Omega) =>
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p) ≤
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p)
```

## Direct retained dependencies

- [AppliedModelingLib.pmfProb](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)

## GJ18InformativeRatingSystems.source\_at\_most\_one\_match\_per\_period

**Declaration location:** paper-local

## Lean declaration body

```
type:
{Seller : Type u_1} → (Seller → ℝ) → Prop

value:
fun {Seller : Type u_1} (sampleRate : Seller → ℝ) =>
∀ (θ : Seller) (k : ℕ),
AppliedModelingLib.Probability.floorSampleCount sampleRate θ (k + 1) =
AppliedModelingLib.Probability.floorSampleCount sampleRate θ k ∨
AppliedModelingLib.Probability.floorSampleCount sampleRate θ (k + 1) =
AppliedModelingLib.Probability.floorSampleCount sampleRate θ k + 1
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.floorSampleCount](#)

## GJ18InformativeRatingSystems.source\_monotone\_match\_rates

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → (Fin n → ℝ) → Prop

value:
fun {n : ℕ} (sampleRate : Fin n → ℝ) => Monotone sampleRate
```

## GJ18InformativeRatingSystems.source\_nonnegative\_match\_rates

**Declaration location:** paper-local

## Lean declaration body

```
type:
{Seller : Type u_1} → (Seller → ℝ) → Prop

value:
fun {Seller : Type u_1} (sampleRate : Seller → ℝ) => ∀ (θ : Seller), 0 ≤ sampleRate θ
```

## GJ18InformativeRatingSystems.source\_match\_schedule\_coherence

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n : ℕ} → (Fin n → ℝ) → Prop

value:
fun {n : ℕ} (sampleRate : Fin n → ℝ) =>
  GJ18InformativeRatingSystems.source_nonnegative_match_rates sampleRate ∧
  GJ18InformativeRatingSystems.source_monotone_match_rates sampleRate ∧
  GJ18InformativeRatingSystems.source_at_most_one_match_per_period sampleRate
```

## Direct retained dependencies

- [GJ18InformativeRatingSystems.source\\_at\\_most\\_one\\_match\\_per\\_period](#)
- [GJ18InformativeRatingSystems.source\\_monotone\\_match\\_rates](#)
- [GJ18InformativeRatingSystems.source\\_nonnegative\\_match\\_rates](#)

## GJ18InformativeRatingSystems.source\_rating\_scores\_in\_unit\_interval

**Declaration location:** paper-local

## Lean declaration body

```
type:
{Rating : Type u_1} → (Rating → ℝ) → Prop

value:
fun {Rating : Type u_1} (score : Rating → ℝ) => ∀ (r : Rating), 0 ≤ score r ∧ score r ≤ 1
```

## GJ18InformativeRatingSystems.source\_strict\_ordinal\_cumulative\_tail\_decrease\_at\_displayed\_cutoffs

**Declaration location:** paper-local

## Lean declaration body

```
type:
{Seller : Type u_1} → {m : ℕ} → AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller (Fin (m + 1)) → Prop

value:
fun {Seller : Type u_1} {m : ℕ} (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller (Fin (m + 1))) =>
  ∀ (θ : Seller) (r : Fin m),
    GJ18InformativeRatingSystems.sourceOrdinalUpperTail M θ r.succ <
    GJ18InformativeRatingSystems.sourceOrdinalUpperTail M θ r.castSucc
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.sourceOrdinalUpperTail](#)

## GJ18InformativeRatingSystems.source\_strict\_ordinal\_quality\_tail\_increase\_above\_bottom

**Declaration location:** paper-local

## Lean declaration body

```
type:
{n m : ℕ} → AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1)) → Prop

value:
fun {n m : ℕ} (M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))) =>
  ∀ (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) (t : Fin (m + 1)),
    0 < ↑t →
      GJ18InformativeRatingSystems.sourceOrdinalUpperTail M (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p)
      t <
      GJ18InformativeRatingSystems.sourceOrdinalUpperTail M (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p)
      t
```

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)
- [GJ18InformativeRatingSystems.sourceOrdinalUpperTail](#)

## GJ18InformativeRatingSystems.source\_strictly\_increasing\_rating\_scores

**Declaration location:** paper-local

### Lean declaration body

```
type:
{Rating : Type u_1} → [Preorder Rating] → (Rating → ℝ) → Prop

value:
fun {Rating : Type u_1} [Preorder Rating] (score : Rating → ℝ) => StrictMono score
```

## GJ18InformativeRatingSystems.FiniteOrdinalSourceModel

**Declaration location:** paper-local

### Lean declaration body

```
field match_schedule:
∀ {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ},
  GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  GJ18InformativeRatingSystems.source_match_schedule_coherence sampleRate

field scores_in_unit_interval:
∀ {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ},
  GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  GJ18InformativeRatingSystems.source_rating_scores_in_unit_interval M.score

field scores_strictly_increasing:
∀ {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ},
  GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  GJ18InformativeRatingSystems.source_strictly_increasing_rating_scores M.score

field tails_strictly_decrease_at_displayed_cutoffs:
∀ {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ},
  GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  GJ18InformativeRatingSystems.source_strict_ordinal_cumulative_tail_decrease_at_displayed_cutoffs M

field quality_tails_strictly_increase_above_bottom:
∀ {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ},
  GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  GJ18InformativeRatingSystems.source_strict_ordinal_quality_tail_increase_above_bottom M
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.source\\_match\\_schedule\\_coherence](#)
- [GJ18InformativeRatingSystems.source\\_rating\\_scores\\_in\\_unit\\_interval](#)
- [GJ18InformativeRatingSystems.source\\_strict\\_ordinal\\_cumulative\\_tail\\_decrease\\_at\\_displayed\\_cutoffs](#)
- [GJ18InformativeRatingSystems.source\\_strict\\_ordinal\\_quality\\_tail\\_increase\\_above\\_bottom](#)
- [GJ18InformativeRatingSystems.source\\_strictly\\_increasing\\_rating\\_scores](#)

## GJ18InformativeRatingSystems.ClarifiedSourceModel.correctedTheorem1Rate

**Declaration location:** paper-local

### Lean declaration body

```
type:
{n m : ℕ} →
  {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} →
  {sampleRate : Fin n → ℝ} → GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate → WithTop ℝ

value:
fun {n m : ℕ} {M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ}
  (model : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate) =>
  GJ18InformativeRatingSystems.minFiniteChainAdjacentThresholdRateTop M sampleRate
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.ClarifiedSourceModel](#)
- [GJ18InformativeRatingSystems.finiteChainAdjacentIndex](#)
- [GJ18InformativeRatingSystems.minFiniteChainAdjacentThresholdRateTop](#)

# Material library prerequisites

## AppliedModelingLib.Probability.floorSampleCount

**Source locator:** cited publication, lines 669-670

### Verbatim paper-source connection

{\bf Rating accumulation}. Sellers accumulate ratings over time by matching with buyers. At each time step, each seller matches with at most a single  
↪ buyer. We make one key assumption that drives the accumulation of ratings: in particular, that sellers of higher quality are more likely to be matched.  
↪ %  
We consider an analysis that is asymptotic in the number of ratings received by sellers and so we model this visibility benefit by assuming that {\em  
↪ sellers of higher quality accumulate ratings at a faster rate}. In particular, we assume the existence of a nondecreasing {\em match function}  
↪  $g(\theta)$ , where a seller of type  $\theta$  receives  $n_k(\theta) = \lfloor k g(\theta) \rfloor$  matches, and thus ratings, up to time  $k$ .

### Lean-expanded library semantic target

```
type:
{Seller : Type u_1} → (Seller → ℝ) → Seller → ℕ → ℕ

value:
fun {Seller : Type u_1} (sampleRate : Seller → ℝ) (θ : Seller) (k : ℕ) => [↑ k * sampleRate θ],
```

### Exact Lean library declaration

```
/-- Paper sample count  $n_k(\theta) = \lfloor k g(\theta) \rfloor$ . -/
def floorSampleCount {Seller : Type*} (sampleRate : Seller → ℝ)
  (θ : Seller) (k : ℕ) : ℕ :=
  Nat.floor ((k : ℝ) * sampleRate θ)
```

### Recorded source-to-library screening Verdict: matches

**Reason:** Matches the literal source formula  $n_k(\theta)=\lfloor k g(\theta) \rfloor$ : the expanded definition is exactly the natural floor of  $k$  times the seller-specific match rate. The source's monotonicity and at-most-one-match discussion is not silently promoted into this formula declaration.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

## AppliedModelingLib.Probability.finiteChainJointFloorAverageScore

**Source locator:** cited publication, lines 697-702

### Verbatim paper-source connection

Let  $y_0(\theta), y_1(\theta), y_2(\theta), \dots$  be the sequence of ratings received by the seller of type  $\theta$ . The (em aggregate score) up to time  $k$  of this seller is the average score from ratings received:

$$x_k(\theta) = \frac{1}{n_k(\theta)} \sum_{\ell=0}^{n_k(\theta)-1} y_\ell(\theta).$$

(We presume  $x_0(\theta) = 0$  for all  $\theta$ .) Since  $\phi(y) \in [0,1]$  for all  $y$ , the score  $x_k$  also lies in  $[0,1]$ .

**Formalized review target** The archival source above is not asserted equivalent to this formalized target.

For each seller type  $\theta$  and horizon  $k$ , if  $n_k(\theta) = \text{floor}(k \cdot g(\theta))$ , the aggregate score averages exactly  $n_k(\theta)$  rating scores over a finite family indexed by  $\text{Fin } n_k(\theta)$ ; the zero-sample convention remains explicit.

**Archival source anchor:** cited publication, lines 697-702

### Lean-expanded library semantic target

```
type:
{n : ℕ} →
{Rating : Type u_1} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating →
(sampleRate : Fin n → ℝ) →
(k : ℕ) → AppliedModelingLib.Probability.finiteChainJointFloorRatingSample Rating sampleRate k → Fin n → ℝ

value:
fun {n : ℕ} {Rating : Type u_1} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ) (k : ℕ)
(sample : AppliedModelingLib.Probability.finiteChainJointFloorRatingSample Rating sampleRate k) (θ : Fin n) =>
(↑ (AppliedModelingLib.Probability.floorSampleCount sampleRate θ k) : ℝ)-1 *
AppliedModelingLib.Probability.finitelidScoreSum M.score (sample θ)
```

### Exact Lean library declaration

```
-- Aggregate score projection from the joint floor-count rating sample. -/
def finiteChainJointFloorAverageScore
{n : ℕ} {Rating : Type*} [Fintype Rating] [DecidableEq Rating]
(M : FiniteRatingLDPMModel (Fin n) Rating) (sampleRate : Fin n → ℝ)
(k : ℕ) (sample : finiteChainJointFloorRatingSample Rating sampleRate k)
(θ : Fin n) : ℝ :=
((floorSampleCount sampleRate θ k : ℕ) : ℝ)-1 *
finitelidScoreSum M.score (sample θ)
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.finiteChainJointFloorRatingSample](#)
- [AppliedModelingLib.Probability.finitelidScoreSum](#)
- [AppliedModelingLib.Probability.floorSampleCount](#)

**Recorded source-to-library screening Verdict:** matches approved corrected target

**Reason:** Compared cited publication:697-702, including the archived inclusive sum from 0 through  $n_k(\theta)$ , with the identity-bound corrected target for GJ18-AGGREGATE-INDEX-01, whose approval record expressly disclaims archival equivalence. The displayed `floorSampleCount` is  $\text{Nat.floor}(k * \text{sampleRate } \theta)$ ; `finiteChainJointFloorRatingSample` supplies exactly  $\text{Fin } (\text{floorSampleCount sampleRate } \theta \text{ } k)$  ratings at each  $\theta$ ; `finitelidScoreSum` sums `M.score` over that entire finite index family; and the reviewed definition multiplies that sum by the real reciprocal of precisely the same count. Thus it averages exactly  $n_k(\theta)$  observations, as required by the corrected target, rather than the archival  $n_k(\theta)+1$  summands. At zero count the index family is empty, the sum is zero, and the totalized real reciprocal yields aggregate score zero, including  $k=0$ . The arbitrary real-valued score field introduces no additional premise and specializes to the source scores in  $[0,1]$ , for which this average also lies in  $[0,1]$ . The displayed `iidFloorSourceStateScore` is a direct application of this same aggregate, and all three displayed source-claim uses pass that score unchanged into their state statistics. Those uses do not change the count, normalization, or observation indexing. This decision covers the corrected finite-chain aggregate projection only; it does not establish the source state evolution or independently certify the downstream rate claims.

**Reviewer check:** ☐ Matches formalized target

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

## AppliedModelingLib.Probability.ratingLogMGF

Source locator: cited publication, lines 754-761

### Verbatim paper-source connection

```
\begin{restatable}{theorem}{lemhatW}
\label{lem:hatW}
\begin{equation}
\label{eq:rW}
r \triangleq - \lim_{k \rightarrow \infty} \frac{1}{k} \log (1 - \{W\}_k) = \min_{\{0 \leq i < M\}} \inf_{a \in \mathbb{R}} \left\{ g(\theta_{i+1}) l(a|\theta_{i+1}) + \right.
\left. \rightarrow g(\theta_i) l(a|\theta_i) \right\}
\end{equation}
where
 $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta) = \log \sum_{y \in Y} \{\rho(\theta, y | Y)\} \exp(z \phi(y))$  is the log
 $\rightarrow$  moment generating function of a single rating given to seller of type  $\theta$ .
```

[Next verbatim source excerpt]

```
\begin{lemma}
 $\lim_{k \rightarrow \infty} - \frac{1}{k} \log \left[ \mu(x_k(\theta_1) - x_k(\theta_2)) \leq 0 \mid \theta_1, \theta_2 \right] = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) l(a|\theta_1) + g(\theta_2) l(a|\theta_2) \right\}$  where  $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ ,  $\Lambda(z|\theta)$  is the
 $\rightarrow$  log moment generating function of a single sample from  $x(\theta_1)$ , and  $g(\theta)$  is the sampling rate.
\end{lemma}
```

[Next verbatim source excerpt]

```
\begin{restatable}{lemma}{lemPkld}
%
\label{lem:Pk_LD}
Given  $\theta_1, \theta_2$ , let  $\{P\}_k(\theta_1, \theta_2) = 1 - P_k(\theta_1, \theta_2)$ . Then:
\begin{equation}
\label{eq:LDrate}
- \lim_{k \rightarrow \infty} \frac{1}{k} \log \{P\}_k(\theta_1, \theta_2) = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) l(a|\theta_1) + g(\theta_2) l(a|\theta_2) \right.
\left. \rightarrow \right\},
\end{equation}
where  $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta)$  is the log moment generating function of a single rating given to
 $\rightarrow$  seller of type  $\theta$ :
 $[\Lambda(z|\theta) = \log \sum_{y \in Y} \rho(\theta, y | Y) \exp(z \phi(y))]$ 
```

### Lean-expanded library semantic target

```
type:
{Seller : Type u_1} →
{Rating : Type u_2} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating → Seller → ℝ → ℝ

value:
fun {Seller : Type u_1} {Rating : Type u_2} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating) (θ : Seller) (z : ℝ) =>
M.logMGF θ z
```

### Exact Lean library declaration

```
/-- Source log-MGF  $\Lambda(z | \theta) = \log \sum_y \rho(\theta, y | Y) \exp(z \phi(y))$ . -/
def ratingLogMGF {Seller Rating : Type*} [Fintype Rating] [DecidableEq Rating]
(M : FiniteRatingLDPMModel Seller Rating) (θ : Seller) (z : ℝ) : ℝ :=
M.logMGF θ z
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel.logMGF](#)

### Recorded source-to-library screening Verdict: matches

**Reason:** Matches the displayed log-MGF definition  $\Lambda(z|\theta) = \log \sum_y \rho(\theta, y|Y) \exp(z \phi(y))$ ; the expanded declaration is the finite rating-model logMGF at the selected seller and real argument.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

### Reviewer annotation

## AppliedModelingLib.Probability.ratingRateFunction

Source locator: cited publication, lines 754-761

### Verbatim paper-source connection

```
\begin{restatable}{theorem}{lemhatW}
\label{lem:hatW}
\begin{equation}
\label{eq:rW}
r \triangleq - \lim_{k \rightarrow \infty} \frac{1}{k} \log (1 - \{W\}_k) = \min_{0 \leq i < M} \inf_{a \in \mathbb{R}} \left\{ g(\theta_{i+1}) I(a|\theta_{i+1}) + \right.
\left. \rightarrow g(\theta_i) I(a|\theta_i) \right\}
\end{equation}
where
 $I(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta) = \log \sum_{y \in Y} \{\rho(\theta, y | Y)\} \exp(z \phi(y))$  is the log
 $\rightarrow$  moment generating function of a single rating given to seller of type  $\theta$ .
```

[Next verbatim source excerpt]

```
\begin{lemma}
 $\lim_{k \rightarrow \infty} \frac{1}{k} \log \left[ \mu(x_k(\theta_1) - x_k(\theta_2)) \leq 0 \mid \theta_1, \theta_2 \right] = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) I(a|\theta_1) + g(\theta_2) I(a|\theta_2) \right\}$  where  $I(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ ,  $\Lambda(z|\theta)$  is the
 $\rightarrow$  log moment generating function of a single sample from  $x(\theta_1)$ , and  $g(\theta)$  is the sampling rate.
\end{lemma}
```

[Next verbatim source excerpt]

```
\begin{restatable}{lemma}{lemPkld}
%
\label{lem:Pk_LD}
Given  $\theta_1, \theta_2$ , let  $\{P\}_k(\theta_1, \theta_2) = 1 - P_k(\theta_1, \theta_2)$ . Then:
\begin{equation}
\label{eq:LDrate}
- \lim_{k \rightarrow \infty} \frac{1}{k} \log \{P\}_k(\theta_1, \theta_2) = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) I(a|\theta_1) + g(\theta_2) I(a|\theta_2) \right.
\left. \rightarrow \right\},
\end{equation}
where  $I(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta)$  is the log moment generating function of a single rating given to
 $\rightarrow$  seller of type  $\theta$ :
 $[\Lambda(z|\theta) = \log \sum_{y \in Y} \rho(\theta, y | Y) \exp(z \phi(y))]$ 
```

### Lean-expanded library semantic target

```
type:
{Seller : Type u_1} →
{Rating : Type u_2} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] → AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating → Seller → ℝ → ℝ

value:
fun {Seller : Type u_1} {Rating : Type u_2} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating) (θ : Seller) (a : ℝ) =>
M.rateFunction θ a
```

### Exact Lean library declaration

```
/-- Source rate function  $I(a | \theta) = \sup_z \{ z a - \Lambda(z | \theta) \}$ . -/
def ratingRateFunction {Seller Rating : Type*} [Fintype Rating]
[DecidableEq Rating]
(M : FiniteRatingLDPMModel Seller Rating) (θ : Seller) (a : ℝ) : ℝ :=
M.rateFunction θ a
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel.rateFunction](#)

### Recorded source-to-library screening Verdict: matches

**Reason:** Matches the archival real-valued Legendre display  $I(a|\theta)=\sup_z \{za-\Lambda(z|\theta)\}$ . This route remains intentionally separate from the support-safe extended-rate correction and makes no claim that the real-valued expression is valid for every threshold.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

## AppliedModelingLib.Probability.ratingRateFunctionTop

Source locator: cited publication, lines 754-761

### Verbatim paper-source connection

```
\begin{restatable}{theorem}{lemhatW}
\label{lem:hatW}
\begin{equation}
\label{eq:rW}
r \triangleq - \lim_{k \rightarrow \infty} \frac{1}{k} \log (1 - \{W\}_k) = \min_{0 \leq i < M} \inf_{a \in \mathbb{R}} \left\{ g(\theta_{i+1}) l(a|\theta_{i+1}) + \right.
\left. \rightarrow g(\theta_i) l(a|\theta_i) \right\}
\end{equation}
where
 $(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta) = \log \sum_{y \in Y} \{\rho(\theta, y | Y)\} \exp(z \phi(y))$  is the log
 $\rightarrow$  moment generating function of a single rating given to seller of type  $\theta$ .
```

[Next verbatim source excerpt]

```
\begin{lemma}
 $\lim_{k \rightarrow \infty} \frac{1}{k} \log \left[ \mu(x_k(\theta_1) - x_k(\theta_2)) \leq 0 \mid \theta_1, \theta_2 \right] = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) l(a|\theta_1) + g(\theta_2) l(a|\theta_2) \right\}$  where  $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ ,  $\Lambda(z|\theta)$  is the
 $\rightarrow$  log moment generating function of a single sample from  $x(\theta_1)$ , and  $g(\theta)$  is the sampling rate.
\end{lemma}
```

[Next verbatim source excerpt]

```
\begin{restatable}{lemma}{lemPkld}
%
\label{lem:Pk_LD}
Given  $\theta_1, \theta_2$ , let  $b\{P\}_k(\theta_1, \theta_2) = 1 - P_k(\theta_1, \theta_2)$ . Then:
\begin{equation}
\label{eq:LDrate}
- \lim_{k \rightarrow \infty} \frac{1}{k} \log b\{P\}_k(\theta_1, \theta_2) = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) l(a|\theta_1) + g(\theta_2) l(a|\theta_2) \right.
\left. \rightarrow \right\},
\end{equation}
where  $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta)$  is the log moment generating function of a single rating given to
 $\rightarrow$  seller of type  $\theta$ :
 $[\Lambda(z|\theta) = \log \sum_{y \in Y} \rho(\theta, y | Y) \exp(z \phi(y))]$ 
```

### Lean-expanded library semantic target

```
type:
{Seller : Type u_1} →
{Rating : Type u_2} →
[inst : Fintype Rating] →
[inst_1 : DecidableEq Rating] →
AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating → Seller → ℝ → WithTop ℝ

value:
fun {Seller : Type u_1} {Rating : Type u_2} [Fintype Rating] [DecidableEq Rating]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel Seller Rating) (θ : Seller) (a : ℝ) =>
M.rateFunctionTop θ a
```

### Exact Lean library declaration

```
/--
Extended source rate function. Thresholds outside the finite score hull have
rate `T`, avoiding the real-valued `sSup` boundary that appears in unrestricted
Legendre transforms.
-/
def ratingRateFunctionTop {Seller Rating : Type*} [Fintype Rating]
[DecidableEq Rating]
(M : FiniteRatingLDPMModel Seller Rating) (θ : Seller) (a : ℝ) :
WithTop ℝ :=
M.rateFunctionTop θ a
```

### Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel.rateFunctionTop](#)

### Recorded source-to-library screening Verdict: matches

**Reason:** Matches the explicitly recorded support-safe corrected rate declaration: the expanded route returns WithTop Real for the same Legendre expression, allowing infinite cost outside finite score support. This is a distinct corrected definition route, not a claim that the archival real-only display already had that codomain.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

### Reviewer annotation

# Paper-specific semantic prerequisites

## GJ18InformativeRatingSystems.ClarifiedSourceModel

### Paper-local declaration:

GJ18InformativeRatingSystems.ClarifiedSourceModel

**Source locator:** cited publication, lines 19-26

### Verbatim paper-source connection

| `GJ18-CORR-SELLER-CARRIER-2026-07-24` | `adjectivesmodel.tex:22-24,85-90,111-125` | Seller types are  $\theta_0 < \dots < \theta_{M-1}$  with  $M \geq 2$  and a uniform prior. | Makes the objective's nonzero denominator and adjacent-pair carrier explicit. |  
| `GJ18-CORR-MATCH-RATES-2026-07-24` | `adjectivesmodel.tex:26-29` | For every type,  $0 < g(\theta) \leq 1$ ;  $n_k(\theta) = \text{floor}(k g(\theta))$ . |  
| Strengthens the displayed nondecreasing rate condition so every comparison has diverging sample size while preserving at most one match per period. |  
| `GJ18-CORR-AGGREGATE-INDEX-2026-07-24` | `adjectivesmodel.tex:54-59` | The aggregate is the average of exactly  $n_k(\theta)$  ratings, indexed for example by  $\ell = 1, \dots, n_k(\theta)$ , with the stated zero-sample convention. | Corrects the displayed off-by-one summation endpoint. |  
| `GJ18-CORR-RATING-CARRIER-2026-07-24` | `adjectivesmodel.tex:32-51` | The ordered rating scale has at least two levels; scores are strictly increasing; within-type tails decrease at displayed cutoffs; cross-type upper tails are strict only above the bottom cutoff. | Excludes the vacuous singleton scale and the impossible strict bottom-tail condition. No terminal full-support clause is added. |  
| `GJ18-CORR-IID-STATE-LAW-2026-07-24` | `adjectivesmodel.tex:40,67-73` | Conditional on seller type, rating draws are iid with law  $\rho(\theta, \cdot | Y)$  and histories are independent across sellers. At horizon  $k$ , the state law is the induced finite product law. | Replaces the archive's informal/ill-typed  $\mu_k$  recurrence as the governing state-evolution clause. |  
| `GJ18-CORR-ADJACENT-INDEX-2026-07-24` | `adjectivesmodel.tex:111-125; appendix\_theory.tex:77-95` | The Theorem 1 minimum ranges over  $0 \leq i \leq M-2$ , exactly the valid adjacent pairs. | Corrects the endpoint that can name nonexistent  $\theta_M$ . |  
| `GJ18-CORR-EXTENDED-RATE-2026-07-24` | `adjectivesmodel.tex:111-125; appendix\_theory.tex:7-8,44-53` |  $I(a | \theta) = \sup_z \{z a - \Lambda(z | \theta)\}$  is extended-real valued. The pairwise and adjacent minima use extended-real arithmetic; the final corrected-model rate is proved finite and then identified with its real representative. | Corrects the codomain required for out-of-support thresholds. |  
| `GJ18-CORR-APPENDIX-DISCRETE-PROOF-2026-07-24` | `appendix\_theory.tex:7-69` | The weak-inversion and  $1-P_k$  rate are proved directly for finite iid product laws, including the fixed-constant comparison. | Replaces the unsupported continuum-integral/Laplace transfer. |

[Next verbatim source excerpt]

## Checked Realization

`ClarifiedSourceModel` is the complete Lean model record for these clauses. It visibly contains the seller-count, uniform-prior, match-rate, rating-scale, ordinal-tail, and iid-state-law fields. `PaperInterface.lean` exposes both the state-level extended-rate theorem and the finite-real-rate theorem for the corrected  $W_k$  objective.

The archived  $\mu_k$  recurrence and unrestricted real-valued rate display are retained only as explicitly recorded archival deltas and diagnostics. No checked theorem claims to derive the corrected iid state law or corrected rate codomain from those archived expressions.

### Lean-elaborated paper signature

field seller\_type\_count\_pos:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  $0 < n$

field at\_least\_two Seller types:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  $2 \leq n$

field type\_prior:

$\{n m : \mathbb{N}\} \rightarrow$   
 $\{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \rightarrow$   
 $\{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\} \rightarrow \text{GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate} \rightarrow \text{PMF (Fin n)}$

field type\_prior\_uniform:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  
self.type\_prior = GJ18InformativeRatingSystems.clarifiedUniformTypePrior n self.seller\_type\_count\_pos

field match\_rate\_bounds:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  
GJ18InformativeRatingSystems.clarifiedMatchRateBounds sampleRate

field nontrivial\_rating\_scale:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  
GJ18InformativeRatingSystems.clarifiedNontrivialRatingScale m

field ordinal\_source\_model:

$\forall \{n m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\}$   
(self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),  
GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate

field stateLaw:

$\{n m : \mathbb{N}\} \rightarrow$   
 $\{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))\} \rightarrow$   
 $\{sampleRate : \text{Fin n} \rightarrow \mathbb{R}\} \rightarrow$   
 $\text{GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate} \rightarrow$   
 $(k : \mathbb{N}) \rightarrow \text{PMF (GJ18InformativeRatingSystems.iidFloorSourceStateCarrier (Fin (m + 1)) sampleRate k)}$

field conditional\_iid\_ratings\_and\_independent\_seller\_histories:

$\forall \{n\ m : \mathbb{N}\} \{M : \text{AppliedModelingLib.Probability.FiniteRatingLDPMModel } (\text{Fin } n) (\text{Fin } (m + 1))\} \{\text{sampleRate} : \text{Fin } n \rightarrow \mathbb{R}\}$   
 (self : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate) (k :  $\mathbb{N}$ ),  
 self.stateLaw k = GJ18InformativeRatingSystems.iidFloorSourceStateLaw M sampleRate k

## Direct retained dependencies

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [GJ18InformativeRatingSystems.FiniteOrdinalSourceModel](#)
- [GJ18InformativeRatingSystems.clarifiedMatchRateBounds](#)
- [GJ18InformativeRatingSystems.clarifiedNontrivialRatingScale](#)
- [GJ18InformativeRatingSystems.clarifiedUniformTypePrior](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateLaw](#)

## Recorded source-to-declaration screening Verdict: matches

**Reason:** Matches the exact governing model record. The expanded structure contains the finite ordered seller carrier with at least two types, uniform prior, positive bounded matching schedule, nontrivial ordinal rating scale and tails, and the finite iid product state-law equality. Its comments and fields explicitly deny archival derivation from the printed recurrence.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

## GJ18InformativeRatingSystems.sourceStatePairwisePk

### Paper-local declaration:

GJ18InformativeRatingSystems.sourceStatePairwisePk

**Source locator:** cited publication, lines 721-725

### Verbatim paper-source connection

Formally, given  $\theta_1 > \theta_2$ , define  $P_k(\theta_1, \theta_2)$  as follows:

```
\begin{equation}
\label{eq:Pk}
P_k(\theta_1, \theta_2) = \mu_k(x_k(\theta_1) > x_k(\theta_2) | \theta_1, \theta_2) - \mu_k(x_k(\theta_1) < x_k(\theta_2) | \theta_1, \theta_2).
\end{equation}
```

### Lean-expanded paper semantic target

```
type:
{n : ℕ} →
{Omega : Type u_1} →
[Fintype Omega] →
[DecidableEq Omega] → PMF Omega → (Omega → Fin n → ℝ) → GJ18InformativeRatingSystems.finiteChainOrderedPair n → ℝ

value:
fun {n : ℕ} {Omega : Type u_1} [Fintype Omega] [DecidableEq Omega] (mu : PMF Omega) (score : Omega → Fin n → ℝ)
(p : GJ18InformativeRatingSystems.finiteChainOrderedPair n) =>
(AppliedModelingLib.pmfProb mu fun (omega : Omega) =>
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p) >
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p)) -
AppliedModelingLib.pmfProb mu fun (omega : Omega) =>
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p) <
score omega (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p)
```

### Direct retained dependencies

- [AppliedModelingLib.pmfProb](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)

**Recorded source-to-declaration screening** **Verdict:** matches

**Reason:** Matches source equation (8): on precisely one strict high/low pair, the expanded definition subtracts the conditional probability of reversed aggregate-score order from the probability of correct strict order. It is deliberately state-level and does not turn the source recurrence into an iid law.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

## GJ18InformativeRatingSystems.sourceStateWk

### Paper-local declaration:

GJ18InformativeRatingSystems.sourceStateWk

**Source locator:** cited publication, lines 728-733

### Verbatim paper-source connection

We consider the problem of maximizing the following objective, a scaled version of Kendall's  $\tau$  rank correlation between the estimated ranking  $\rightarrow$  of sellers and the true ranking:

$$W_k = \frac{2}{M(M-1)} \sum_{\theta_1 > \theta_2 \in \Theta} P_k(\theta_1, \theta_2)$$

The coefficient ensures that  $W_k$  remains bounded even as  $M$  increases. This objective depends on the model primitives  $R$  (rater behavior) and  $g$  (matching rates), as well as the platform's decisions  $Y$  (levels) and  $\phi$  (score).

### Lean-expanded paper semantic target

type:  
 $\{n : \mathbb{N}\} \rightarrow \{\Omega : \text{Type } u_1\} \rightarrow [\text{Fintype } \Omega] \rightarrow [\text{DecidableEq } \Omega] \rightarrow \text{PMF } \Omega \rightarrow (\Omega \rightarrow \text{Fin } n \rightarrow \mathbb{R}) \rightarrow \mathbb{R}$

value:  
fun {n : ℕ} {Ω : Type u<sub>1</sub>} [Fintype Ω] [DecidableEq Ω] (mu : PMF Ω) (score : Ω → Fin n → ℝ) =>  
2 / (↑n \* (↑n - 1)) \*  
Σ p : GJ18InformativeRatingSystems.finiteChainOrderedPair n,  
GJ18InformativeRatingSystems.sourceStatePairwisePk mu score p

### Direct retained dependencies

- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.sourceStatePairwisePk](#)

### Recorded source-to-declaration screening Verdict: matches

**Reason:** Matches source equation (9): the expanded definition uses the displayed coefficient  $2/(M(M-1))$  and sums the source  $P_k$  objective over exactly the strict ordered high/low pair carrier. No additional model or convergence claim is hidden in this formula declaration.

**Reviewer check:** ☐ Matches source input

*If left unchecked, explain the discrepancy or uncertainty below.*

### Reviewer annotation

# Source claims

## Review row 1

**Source locator:** cited publication, lines 1633-1635

### Verbatim source input

```
\begin{lemma}
  $$ \lim_{k \rightarrow \infty} -\frac{1}{k} \log \left[ \mu(x_k(\theta_1) - x_k(\theta_2)) \leq 0 \mid \theta_1, \theta_2 \right] = \inf_{a \in \mathcal{R}} \left\{ \right. \\
  \rightarrow g(\theta_1) I(a|\theta_1) + g(\theta_2) I(a|\theta_2) \left. \right\} where $I(a|\ell) = \sup_z \{ z a - \Lambda(z|\ell) \}$, $\Lambda(z|\ell)$ is the \\
  \rightarrow \log$ moment generating function of a single sample from $x(\theta_1)$, and $g(\theta)$ is the sampling rate. \label{lem:problesth}
\end{lemma}
```

[Next verbatim source excerpt]

{\bf Sellers}. The system consists of a unit mass of sellers, each associated with a quality  $\theta$ , which is (initially) unknown to the platform. We assume  $\theta$  is drawn independently and uniformly at random from a finite and totally ordered set  $\Theta$ , with  $|\Theta| = M$ . We use  $\theta_i$  to denote the  $i$ th element of  $\Theta$  within this order, for  $0 \leq i < M$ .

[Next verbatim source excerpt]

Formally, given  $\theta_1 > \theta_2$ , define  $P_k(\theta_1, \theta_2)$  as follows:

```
\begin{equation}
\label{eq:Pk}
P_k(\theta_1, \theta_2) = \mu(x_k(\theta_1) > x_k(\theta_2) \mid \theta_1, \theta_2) - \mu(x_k(\theta_1) < x_k(\theta_2) \mid \theta_1, \theta_2).
\end{equation}
```

This expression captures the "errors" made by the ranking according to observed score. In particular, when  $\theta_1 > \theta_2$  but  $x_k(\theta_1) < x_k(\theta_2)$ , the aggregate score ranking swaps the ordering of sellers  $\theta_1$  and  $\theta_2$ . Thus, a good rating system has large  $P_k(\theta_1, \theta_2)$ .

We consider the problem of maximizing the following objective, a scaled version of {Kendall's  $\tau$  rank correlation} between the estimated ranking of sellers and the true ranking:

```
\begin{equation}
\label{eq:objective}
W_k = \frac{1}{2} \sum_{\theta_1 > \theta_2 \in \Theta} P_k(\theta_1, \theta_2)
\end{equation}
```

[Next verbatim source excerpt]

$$\begin{aligned} &= - \lim_{k \rightarrow \infty} \frac{1}{k} \log \left( 1 - \frac{1}{M(M-1)} \sum_{\theta_1 > \theta_2 \in \Theta} P_k(\theta_1, \theta_2) \right) \\ &= - \lim_{k \rightarrow \infty} \frac{1}{k} \log \frac{1}{M(M-1)} \sum_{0 \leq i < j \leq M} \mathbb{P}_k(\theta_j, \theta_i) \\ &= - \max_{0 \leq i < j \leq M} \left( \lim_{k \rightarrow \infty} \frac{1}{k} \log \left( \mathbb{P}_k(\theta_j, \theta_i) \right) \right) \text{\label{eqnstep:ldpproperty}} \\ &= \min_{0 \leq i < j \leq M} \left( - \lim_{k \rightarrow \infty} \frac{1}{k} \log \left( \mathbb{P}_k(\theta_j, \theta_i) \right) \right) \\ &= \min_{0 \leq i < j \leq M} \inf_{a \in \mathcal{R}} \left\{ g(\theta_j) I(a|\theta_j) + g(\theta_i) I(a|\theta_i) \right\} \end{aligned}$$

**Formalized review target** The archival source above is not asserted equivalent to this formalized target.

For every ordered high/low seller pair in the finite ordinal IID model, with positive match rates, a nontrivial rating scale, corrected ordinal tails, and an extended-real rate convention, the source-state weak-inversion probability has an exponential-rate certificate with a finite real representative of the support-safe pairwise threshold rate.

**Archival source anchor:** cited publication, lines 1633-1635

## Expanded PaperInterface specification

```
∀ {n m : ℕ} [DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))) (sampleRate : Fin n → ℝ),
GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
  (∀ (theta : Fin n), 0 < sampleRate theta) →
    1 ≤ m →
      ∀ (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n),
        ∃ (rate : ℝ),
          AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop M sampleRate
            (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p)
            (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p) =
            ↑rate ∧
            AppliedModelingLib.Probability.ExponentialRateCertificate
              (fun (k : ℕ) =>
                GJ18InformativeRatingSystems.sourceStateScoreGapLeftTailProb
                  (GJ18InformativeRatingSystems.iidFloorSourceStateLaw M sampleRate k)
                  (GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k) p)
                rate
```

## Retained governing declarations

- [AppliedModelingLib.Probability.ExponentialRateCertificate](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.floorSampleCount](#)
- [AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop](#)
- [GJ18InformativeRatingSystems.FiniteOrdinalSourceModel](#)

- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateLaw](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateScore](#)
- [GJ18InformativeRatingSystems.sourceStateScoreGapLeftTailProb](#)

## Lean-elaborated claim roles

```

1. parameter: ℕ
2. parameter: ℕ
3. parameter: DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)
4. parameter: AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))
5. parameter: Fin n → ℝ
6. assumption: GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate
7. assumption: ∀ (theta : Fin n), 0 < sampleRate theta
8. assumption: 1 ≤ m
9. parameter: GJ18InformativeRatingSystems.finiteChainOrderedPair n
10. conclusion: ∃ (rate : ℝ),
  AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop M sampleRate
    (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p)
    (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p) =
    ↑rate ∧
  AppliedModelingLib.Probability.ExponentialRateCertificate
    (fun (k : ℕ) =>
      GJ18InformativeRatingSystems.sourceStateScoreGapLeftTailProb
        (GJ18InformativeRatingSystems.iidFloorSourceStateLaw M sampleRate k)
        (GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k) p)
    rate

```

## Lean proof endpoint (built separately)

GJ18InformativeRatingSystems.PaperInterface.source\_appendix\_problessthan\_score\_gap\_rate\_iid\_completion

**Recorded source-to-Spec screening Verdict:** matches\_approved\_corrected\_target

**Reason:** Matches the documented corrected Appendix target: the fully expanded Spec quantifies over the same finite strict high/low pair carrier and certifies the support-safe finite rate for the literal weak score-gap event under the approved finite ordinal IID model. The added model premises and extended-rate convention are exactly the recorded correction; the target does not conflate them with the archival recurrence or real-only infimum.

**Reviewer check:** ☐ Matches formalized target

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

# Review row 2

**Source locator:** cited publication, lines 1670-1681

## Verbatim source input

```
\begin{restatable}{lemma}{lemPkId}
%
\label{lem:Pk_LD}
Given  $\theta_1, \theta_2$ , let  $b_{P_k}(\theta_1, \theta_2) = 1 - P_k(\theta_1, \theta_2)$ . Then:
\begin{equation}
\label{eq:LDrate}
- \lim_{k \rightarrow \infty} \frac{1}{k} \log b_{P_k}(\theta_1, \theta_2) = \inf_{a \in \mathcal{R}} \left\{ g(\theta_1) l(a|\theta_1) + g(\theta_2) l(a|\theta_2) \right.
\left. \right\},
\end{equation}
where  $l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\Lambda(z|\theta)$  is the log moment generating function of a single rating given to
\leftarrow seller of type  $\theta$ :
\left[ \Lambda(z|\theta) = \log \sum_{y \in Y} \rho(\theta, y | Y) \exp(z \phi(y)) \right]
%
\end{restatable}
```

[Next verbatim source excerpt]

**Sellers**. The system consists of a unit mass of sellers, each associated with a quality  $\theta$ , which is (initially) unknown to the platform. We assume  $\theta$  is drawn independently and uniformly at random from a finite and totally ordered set  $\Theta$ , with  $|\Theta| = M$ . We use  $\theta_i$  to denote the  $i$ th element of  $\Theta$  within this order, for  $0 \leq i < M$ .

[Next verbatim source excerpt]

Formally, given  $\theta_1 > \theta_2$ , define  $P_k(\theta_1, \theta_2)$  as follows:

```
\begin{equation}
\label{eq:Pk}
P_k(\theta_1, \theta_2) = \mu_k(x_k(\theta_1) > x_k(\theta_2)|\theta_1, \theta_2) - \mu_k(x_k(\theta_1) < x_k(\theta_2)|\theta_1, \theta_2).
\end{equation}
```

This expression captures the "errors" made by the ranking according to observed score. In particular, when  $\theta_1 > \theta_2$  but  $x_k(\theta_1) < x_k(\theta_2)$ , the aggregate score ranking swaps the ordering of sellers  $\theta_1$  and  $\theta_2$ . Thus, a good rating system has large  $P_k(\theta_1, \theta_2)$ .

We consider the problem of maximizing the following objective, a scaled version of Kendall's  $\tau$  rank correlation between the estimated ranking of sellers and the true ranking:

```
\begin{equation}
\label{eq:objective}
W_k = \frac{1}{M(M-1)} \sum_{\theta_1 > \theta_2 \in \Theta} P_k(\theta_1, \theta_2)
\end{equation}
```

[Next verbatim source excerpt]

$$\begin{aligned} &= - \lim_{k \rightarrow \infty} \frac{1}{k} \log \left( 1 - \frac{1}{M(M-1)} \sum_{\theta_1 > \theta_2 \in \Theta} P_k(\theta_1, \theta_2) \right) \\ &= - \lim_{k \rightarrow \infty} \frac{1}{k} \log \frac{1}{M(M-1)} \sum_{0 \leq i < j \leq M} b_{P_k}(\theta_j, \theta_i) \\ &= - \max_{0 \leq i < j \leq M} \left( \lim_{k \rightarrow \infty} \frac{1}{k} \log \left( b_{P_k}(\theta_j, \theta_i) \right) \right) \quad \text{\label{eqnstep:ldpproperty}} \\ &= \min_{0 \leq i < j \leq M} \inf_{a \in \mathcal{R}} \left\{ g(\theta_j) l(a|\theta_j) + g(\theta_i) l(a|\theta_i) \right\} \end{aligned}$$

**Formalized review target** The archival source above is not asserted equivalent to this formalized target.

For every ordered high/low seller pair in the finite ordinal IID model, with positive match rates, a nontrivial rating scale, corrected ordinal tails, and an extended-real rate convention, one minus the source-state  $P_k$  objective has an exponential-rate certificate with the same finite real representative of the support-safe pairwise threshold rate.

**Archival source anchor:** cited publication, lines 1670-1681

## Expanded PaperInterface specification

```
∀ {n m : ℕ} [DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)]
(M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))) (sampleRate : Fin n → ℝ),
GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate →
(∀ (theta : Fin n), 0 < sampleRate theta) →
1 ≤ m →
∀ (p : GJ18InformativeRatingSystems.finiteChainOrderedPair n),
∃ (rate : ℝ),
AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop M sampleRate
(GJ18InformativeRatingSystems.finiteChainOrderedPairHi p)
(GJ18InformativeRatingSystems.finiteChainOrderedPairLo p) =
↑ rate ∧
AppliedModelingLib.Probability.ExponentialRateCertificate
(fun (k : ℕ) =>
1 -
GJ18InformativeRatingSystems.sourceStatePairwisePk
(GJ18InformativeRatingSystems.iidFloorSourceStateLaw M sampleRate k)
(GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k) p)
rate
```

## Retained governing declarations

- [AppliedModelingLib.Probability.ExponentialRateCertificate](#)
- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)

- [AppliedModelingLib.Probability.floorSampleCount](#)
- [AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop](#)
- [GJ18InformativeRatingSystems.FiniteOrdinalSourceModel](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairHi](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPairLo](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateLaw](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateScore](#)
- [GJ18InformativeRatingSystems.sourceStatePairwisePk](#)

## Lean-elaborated claim roles

```

1. parameter: ℕ
2. parameter: ℕ
3. parameter: DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)
4. parameter: AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))
5. parameter: Fin n → ℝ
6. assumption: GJ18InformativeRatingSystems.FiniteOrdinalSourceModel M sampleRate
7. assumption: ∀ (theta : Fin n), 0 < sampleRate theta
8. assumption: 1 ≤ m
9. parameter: GJ18InformativeRatingSystems.finiteChainOrderedPair n
10. conclusion: ∃ (rate : ℝ),
  AppliedModelingLib.Probability.pairwiseSellerThresholdRateTop M sampleRate
    (GJ18InformativeRatingSystems.finiteChainOrderedPairHi p)
    (GJ18InformativeRatingSystems.finiteChainOrderedPairLo p) =
    ↑ rate ∧
  AppliedModelingLib.Probability.ExponentialRateCertificate
    (fun (k : ℕ) =>
      1 -
        GJ18InformativeRatingSystems.sourceStatePairwisePk
          (GJ18InformativeRatingSystems.iidFloorSourceStateLaw M sampleRate k)
          (GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k) p)
    rate

```

## Lean proof endpoint (built separately)

GJ18InformativeRatingSystems.PaperInterface.source\_appendix\_pk\_id\_complement\_rate\_iid\_completion

### Recorded source-to-Spec screening Verdict: matches approved corrected target

**Reason:** Matches the documented corrected Appendix target: the fully expanded Spec quantifies only over the finite strict high/low pair carrier, retains the finite ordinal and positive-rate/nontrivial-scale premises, and certifies the finite representative of the support-safe pairwise rate for one minus the source-state  $P_k$  objective. The source's omitted pair qualifier and real-only rate display are explicit approved corrections, not hidden derivations.

**Reviewer check:** ☐ Matches formalized target

*If left unchecked, explain the discrepancy or uncertainty below.*

### Reviewer annotation

# Review row 3

**Source locator:** cited publication, lines 754-764

## Verbatim source input

```
\begin{restatable}{theorem}{lemhatW}
\label{lem:hatW}
\begin{equation}
\label{eq:rW}
r \triangleq - \lim_{k \rightarrow \infty} \frac{1}{k} \log (1 - \{W\}_k) = \min_{0 \leq i < M} \inf_{a \in \mathbb{R}} \left\{ g(\theta_{i+1}) l(a|\theta_{i+1}) + \right.
\left. \leftrightarrow g(\theta_i) l(a|\theta_i) \right\}
\end{equation}
where
 $\$l(a|\theta) = \sup_z \{ z a - \Lambda(z|\theta) \}$ , and  $\$\Lambda(z|\theta) = \log \sum_y \{\rho(\theta, y | Y)\} \exp(z \phi(y))$  is the log
 $\leftrightarrow$  moment generating function of a single rating given to seller of type  $\theta$ .
%
%
\end{restatable}
```

**Formalized review target** The archival source above is not asserted equivalent to this formalized target.

For the corrected finite ordinal IID model with at least two uniformly distributed seller types, positive at-most-one-per-period match rates, a nontrivial  
 $\leftrightarrow$  ordinal rating scale, and valid adjacent pairs  $0 \leq i \leq M-2$ , one minus the state-level  $W_k$  objective has a finite real exponential rate equal to the  
 $\leftrightarrow$  finite representative of the support-safe adjacent-pair rate.

**Archival source anchor:** cited publication, lines 754-764

## Expanded PaperInterface specification

```
∀ {n m : ℕ} [DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)]
{M : AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))} {sampleRate : Fin n → ℝ}
(model : GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate),
∃ (rate : ℝ),
model.correctedTheorem1Rate = ↑rate ∧
AppliedModelingLib.Probability.HasExponentialRate
(fun (k : ℕ) =>
1 -
GJ18InformativeRatingSystems.sourceStateWk (model.stateLaw k)
(GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k))
rate
```

## Retained governing declarations

- [AppliedModelingLib.Probability.FiniteRatingLDPMModel](#)
- [AppliedModelingLib.Probability.HasExponentialRate](#)
- [AppliedModelingLib.Probability.floorSampleCount](#)
- [GJ18InformativeRatingSystems.ClarifiedSourceModel](#)
- [GJ18InformativeRatingSystems.ClarifiedSourceModel.correctedTheorem1Rate](#)
- [GJ18InformativeRatingSystems.finiteChainOrderedPair](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateCarrier](#)
- [GJ18InformativeRatingSystems.iidFloorSourceStateScore](#)
- [GJ18InformativeRatingSystems.sourceStateWk](#)

## Lean-elaborated claim roles

```
1. parameter: ℕ
2. parameter: ℕ
3. parameter: DecidableEq (GJ18InformativeRatingSystems.finiteChainOrderedPair n)
4. parameter: AppliedModelingLib.Probability.FiniteRatingLDPMModel (Fin n) (Fin (m + 1))
5. parameter: Fin n → ℝ
6. parameter: GJ18InformativeRatingSystems.ClarifiedSourceModel M sampleRate
7. conclusion: ∃ (rate : ℝ),
model.correctedTheorem1Rate = ↑rate ∧
AppliedModelingLib.Probability.HasExponentialRate
(fun (k : ℕ) =>
1 -
GJ18InformativeRatingSystems.sourceStateWk (model.stateLaw k)
(GJ18InformativeRatingSystems.iidFloorSourceStateScore M sampleRate k))
rate
```

## Lean proof endpoint (built separately)

GJ18InformativeRatingSystems.PaperInterface.author\_approved\_corrected\_model\_theorem1\_finite\_real\_rate

**Recorded source-to-Spec screening Verdict:** matches\_approved\_corrected\_target

**Reason:** Matches the documented corrected target: the fully expanded Spec takes the complete ClarifiedSourceModel and concludes that its support-safe adjacent-pair rate has a finite real representative governing the exponential rate of one minus the state-level  $W_k$  objective. This is exactly the approved finite-chain/IID replacement and does not claim that the archive's recurrence or unrestricted Real display derives it.

**Reviewer check:** ☐ Matches formalized target

*If left unchecked, explain the discrepancy or uncertainty below.*

**Reviewer annotation**

|  |
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|--|